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Level-four approximation to the tachyon potential in superstring field theory

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Abstract: We compute the tachyon potential to level 4 in NS superstring field theory. We obtain 89% of the conjectured vacuum energy.

Keywords: D-branes, Superstring Vacua.

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Contents

1. Introduction

It has been conjectured by Sen that at the stationary point of the tachyon potential for the D-brane-anti-D-brane pair or for the non-BPS D-brane of superstring theories, the negative energy density precisely cancels the brane tensions [1, 2, 3].

For the D-brane of bosonic string theory, this conjecture has been verified by Sen and Zwiebach [4] starting from Witten's open string field theory [5]. Using a level truncation method which was proposed by Kostelecky and Samuel [6] and including fields up to level 4, they found a contribution of 99% of the expected value. Subsequently Taylor and Moeller continued the calculation to level 10 fields and verified the conjecture to $99,9\%$ [7].

In the superstring case the first calculation was done by Berkovits [9] using his Wess-Zumino-Witten like proposal for the string field theory action [10, 11]. The pure tachyon contribution was found to amount to 60% of the conjectured value. This computation was then continued to higher levels by the combined force of Berkovits, Sen and Zwiebach [12]. They included fields up to level 3/2 and got 85% of the expected answer.

In this paper we perform a further check on the conjecture: we continue the calculation retaining fields up to level 2 and get 89% of the conjectured value.

2. Berkovits' superstring field theory

Using the embedding of the $N=1$ superstring into a critical $N=2$ theory found in [13], Berkovits proposed a superstring field theory based on a Wess-Zumino-Witten type action [10, 11]. With slight modifications, this action can be used to describe the NS-sector excitations of a non-BPS brane [9] (the modification needed to include the R-sector fields is as yet unknown). In this section we briefly review the action and some of its properties. This section summarizes parts of [12] relevant to the problem at hand with some additional comments.

A string field describing an NS excitation on a non-BPS D-brane can be represented by a vertex operator of the form

$$\widehat{\Phi} = \Phi \otimes (I \text{ or } \sigma_1) \tag{2.1}$$

where Φ is an operator in the conformal field theory of the NS superstring with the superghost system bosonized as $\beta = \partial\xi e^{-\phi}$ and $\gamma = \eta e^{\phi}$ [15].[1] Φ is restricted to have ghost number 0, picture number 0 and to live in the 'large' Hilbert space which includes the ξ zero mode. The string field $\widehat{\Phi}$ should include states of both the GSO-projected and GSO-unprojected sectors. Fields in the GSO-unprojected sector are tensored with σ_1 and the fields in the GSO-projected sector are tensored with I.

One further defines $\widehat{\eta}_0 = \eta_0 \otimes \sigma_3$ where η_0 is the zero-mode of the η-field, and $\widehat{Q}_B = Q_B \otimes \sigma_3$ where Q_B is the BRST-charge

$$Q_B = \oint dz j_B(z) = \oint dz \Big\{ c(T_m + T_{\xi\eta} + T_\phi) + c\partial c b + \eta\, e^{\phi}\, G_m - \eta\partial\eta e^{2\phi} b \Big\}\,, \tag{2.2}$$

and

$$T_{\xi\eta} = \partial\xi\,\eta, \quad T_\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi\,, \tag{2.3}$$

T_m is the matter stress tensor and G_m is the matter supercurrent. The string field action for the non-BPS D-brane then takes the following form:

$$S = \frac{1}{4g^2}\Big\langle\Big\langle (e^{-\widehat{\Phi}}\widehat{Q}_B e^{\widehat{\Phi}})(e^{-\widehat{\Phi}}\widehat{\eta}_0 e^{\widehat{\Phi}}) - \int_0^1 dt (e^{-t\widehat{\Phi}}\partial_t e^{t\widehat{\Phi}})\{(e^{-t\widehat{\Phi}}\widehat{Q}_B e^{t\widehat{\Phi}}), (e^{-t\widehat{\Phi}}\widehat{\eta}_0 e^{t\widehat{\Phi}})\}\Big\rangle\Big\rangle. \tag{2.4}$$

With the double brackets we mean the following:

$$\langle\langle \widehat{A}_1 \cdots \widehat{A}_n \rangle\rangle = (\text{Trace of matrices}) \Big\langle f_1^{(n)} \circ A_1(0) \cdots f_n^{(n)} \circ A_n(0) \Big\rangle\,. \tag{2.5}$$

The $f_k^{(n)}$ entering in the correlator on the right hand side denote some appropriate conformal transformations [14]. They are defined by

$$f_k^{(n)}(z) = e^{\frac{2\pi i(k-1)}{n}} \left(\frac{1+iz}{1-iz}\right)^{2/n}\,, \qquad \text{for } n \geq 1\,, \tag{2.6}$$

they map the unit circle to wedge-formed pieces of the complex-plane.

[1] We adopt the convention that $e^{q\phi}$ is a fermion for odd values of q, i.e. it anticommutes not only with $e^{q'\phi}$ for odd q' but also with the other fermionic fields in the theory.

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One can show that the action (2.4) is invariant under the gauge transformation

$$\delta e^{\widehat{\Phi}} = \left(\widehat{Q}_B\widehat{\Omega}\right) e^{\widehat{\Phi}} + e^{\widehat{\Phi}} \left(\widehat{\eta}_0\widehat{\Omega}'\right), \tag{2.7}$$

where $\widehat{\Omega}$ and $\widehat{\Omega}'$ are independent gauge parameters. This gauge invariance can be fixed[2] by imposing

$$b_0|\text{State}\rangle = 0 \text{ and } \xi_0|\text{State}\rangle = 0\,. \tag{2.8}$$

In the calculation of the tachyon potential, we can restrict the string field to lie in a subspace $\mathcal{H}_1$ formed by acting only with modes of the stress-energy tensor, the supercurrent and the ghost fields b, c, η, ξ, ϕ, since the other excitations can be consistently put to zero. Furthermore, when restricted to fields lying in $\mathcal{H}_1$ the action has a Z_2 twist invariance under which the fields in the GSO-odd sector carry charge $(-)^{h+1}$ and the fields in the GSO-even sector carry charge $(-)^{h+1/2}$, h is the conformal weight. In the computation of the tachyon potential we can therefore further restrict ourselves to the twist even fields.[3]

The non-polynomial action (2.4) should be formally expanded in the string field $\widehat{\Phi}$, and each term should be accompanied by the appropriate conformal transformations. However, because we will only compute the interactions between a finite number of fields, it is easy to see that one does not need all the terms in the action. The conformal field theory correlators in the action (2.4) are nonvanishing only if the total $(b,\ c)$ number is 3, the $(\eta,\ \xi)$ number is 1 and the total ϕ-charge is -2. In the following we will need only the terms in the action involving up to 6 string fields.

Making use of the cyclicity properties derived in the appendix of [12], the action to this order can be written as

$$\begin{aligned} S = \frac{1}{2g^2}\bigg\langle\!\!\bigg\langle &\frac{1}{2}(\widehat{Q}_B\widehat{\Phi})(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{3}(\widehat{Q}_B\widehat{\Phi})\widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi}) + \frac{1}{12}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi}) - \widehat{\Phi}(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}\Big) + \\ &+ \frac{1}{60}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^3(\widehat{\eta}_0\widehat{\Phi}) - 3\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}\Big) + \\ &+ \frac{1}{360}(\widehat{Q}_B\widehat{\Phi})\Big(\widehat{\Phi}^4(\widehat{\eta}_0\widehat{\Phi}) - 4\widehat{\Phi}^3(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi} + 3\widehat{\Phi}^2(\widehat{\eta}_0\widehat{\Phi})\widehat{\Phi}^2\Big)\bigg\rangle\!\!\bigg\rangle\,. \end{aligned}$$

3. The fields up to level 2

Taking all this together we get the list of contributing fields up to level 2 (table 1). The level of a field is just the conformal weight shifted by 1/2. In this way the tachyon is a level 0 field. We use the notation $|q\rangle$ for the state corresponding with the operator $e^{q\phi}$.

[2]This is a reachable gauge choice (if $L_0 \neq 0$) but we have not been able to prove that it fixes the gauge freedom completely.

[3]This restriction projects out the only state with $L_0 = 0$, namely $\xi\partial\xi\ c\partial c\ e^{-2\phi}$.

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Level	state	vertex operator
0	$\xi_0 c_1 \vert -1\rangle$	$T = \xi c e^{-\phi}$
3/2	$2c_1 c_{-1}\xi_0\xi_{-1}\vert -2\rangle$	$A = c\partial^2 c\xi\partial\xi\ e^{-2\phi}$
	$\xi_0\eta_{-1}\vert 0\rangle$	$E = \xi\eta$
	$\xi_0 c_1 G^m_{-3/2}\vert -1\rangle$	$F = \xi c G^m\ e^{-\phi}$
2	$\xi_0 c_1 \left[(\phi_{-1})^2 - \phi_{-2}\right]\vert -1\rangle$	$K = \xi c\ \partial^2 \left(e^{-\phi}\right)$
	$\xi_0 c_1 \phi_{-2}\vert -1\rangle$	$L = \xi c\ \partial^2\phi\ e^{-\phi}$
	$\xi_0 c_1 L^m_{-2}\vert -1\rangle$	$M = \xi c T^m\ e^{-\phi}$
	$2\xi_0 c_{-1}\vert -1\rangle$	$N = \xi\partial^2 c\ e^{-\phi}$
	$\xi_0\xi_{-1}\eta_{-1}c_1\vert -1\rangle$	$P = \xi\partial\xi\eta c\ e^{-\phi}$

Table 1: The contributing states up to level two and their corresponding vertex operators.

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The level 0 and level 2 fields should be tensored with σ_1 and the level 3/2 fields with I.

We list the conformal transformations of the fields in appendix A.

4. The tachyon potential

We have calculated the tachyon potential involving fields up to level 2, including only the terms up to level 4 (the level of a term in the potential is defined to be the sum of the levels of the fields entering into it). We have performed the actual computation in the following way. The conformal tranformations of the fields were calculated by hand. The computation of all the correlation functions between these transformed fields was done with the help of Mathematica: we have written a program to compute the necessary CFT correlation functions. We have performed an extra check by calculating some of the correlators on the upper half plane instead of the disc. Denoting

$$\widehat{\Phi} = t\widehat{T} + a\widehat{A} + e\widehat{E} + f\widehat{F} + k\widehat{K} + l\widehat{L} + m\widehat{M} + n\widehat{N} + p\widehat{P} \qquad (4.1)$$

we give the result with coefficients evaluated numerically up to 6 significant digits (subscripts refer to the level)

$$\begin{aligned}
V(\widehat{\Phi}) &= -S(\widehat{\Phi}) \\
&= V_0 + V_{3/2} + V_2 + V_3 + V_{7/2} + V_4 \\
V(\widehat{\Phi})_0 &= 2\pi^2 M(-0.25t^2 + 0.5t^4) \\
V(\widehat{\Phi})_{3/2} &= 2\pi^2 M(-at^2 - 0.25et^2 - 0.518729et^4) \\
V(\widehat{\Phi})_2 &= 2\pi^2 M\Big(0.333333kt^3 + 1.83333lt^3 - 3.75mt^3 + 2.83333nt^3 + 0.25pt^3\Big)
\end{aligned}$$

$$
\begin{aligned}
V(\widehat{\Phi})_3 &= 2\pi^2 M\Big(2ae + 5f^2 + 4.96405aet^2 - 0.66544e^2t^2 + \\
&\quad + 5.47589eft^2 + 5.82107f^2t^2 + 0.277778e^2t^4\Big) \\
V(\widehat{\Phi})_{7/2} &= 2\pi^2 M\Big(-3.03704akt - 7.11111alt + 2.77778amt - 1.62963ant - \\
&\quad - 1.55556apt + 0.12963ekt - 0.296296elt + 0.694444emt - \\
&\quad - 1.2963ent + 0.944444ept - 11.8519flt - 8.88889fmt - \\
&\quad - 2.96296fpt - 2.87299ekt^3 - 1.94348elt^3 + 4.35732emt^3 - \\
&\quad - 4.77364ent^3 + 0.605194ept^3\Big) \\
V(\widehat{\Phi})_4 &= 2\pi^2 M\Big(3k^2 - 3kl + 1.5l^2 + 5.625m^2 - 3n^2 - 0.75p^2 + 10.3958k^2t^2 + \\
&\quad + 0.791667klt^2 - 1.875kmt^2 + 5.54167knt^2 - 1.4375kpt^2 + \\
&\quad + 6.70833l^2t^2 - 10.3125lmt^2 + 11.9167lnt^2 - 0.875lpt^2 + \\
&\quad + 14.7656m^2t^2 - 15.9375mnt^2 - 1.40625mpt^2 + \\
&\quad + 5.83333n^2t^2 - 0.5npt^2 - 1.5p^2t^2\Big)\,.
\end{aligned}
$$

Our results for V_0, $V_{3/2}$ and V_3 agree completely with [12][4].

This potential has extrema at $(\pm t_0, a_0, e_0, f_0, \pm k_0, \pm l_0, \pm m_0, \pm n_0, \pm p_0)$ with

$$
\begin{aligned}
t_0 &= 0.602101 \\
a_0 &= 0.0521934\,, & e_0 &= 0.0430366\,, & f_0 &= -0.0138164\,, \\
k_0 &= -0.01019\,, & l_0 &= -0.0450433\,, & m_0 &= 0.0322127\,, \\
n_0 &= 0.0473113\,, & p_0 &= 0.021291\,.
\end{aligned}
$$

At these extrema

$$
V = -0.891287\,M\,, \tag{4.2}
$$

so we see that at the level 4 we get 89% of the conjectured value $V = -M$ (M is the D-brane mass). In the potential computed to level 4 all the fields but t appear only quadratically. They can be integrated out very easily to give the effective potential $V(t)$ see figure 1.

5. Conclusions and outlook

The last few months have seen an increased confidence in string field theory as a calculational tool for doing off-shell string calculations. In this letter, we have used Berkovits' superstring field theory to calculate the tachyon potential up to level four. Our result shows a further convergence towards the total vacuum energy conjectured by Sen, albeit less rapid than the contributions of the previous levels. Therefore it would be interesting to persue the calculation to higher levels [16]. At present, not

[4] Note that our field F is defined with a different sign from the one in [12]

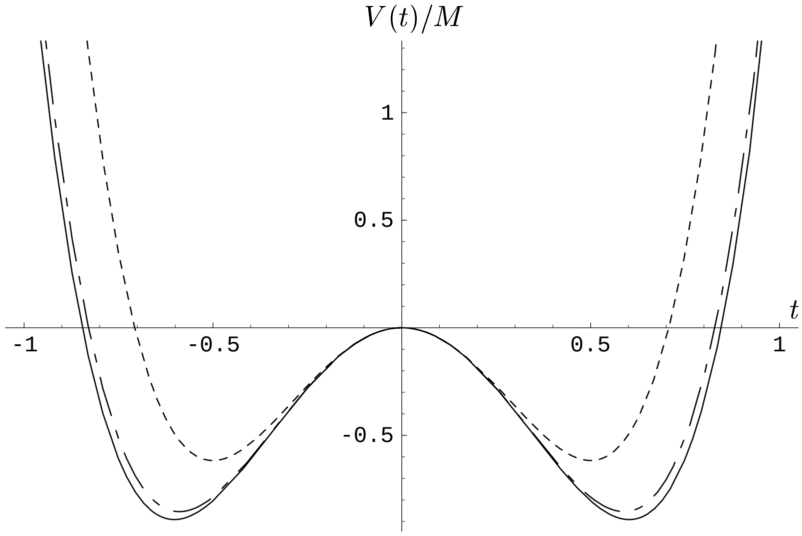

Figure 1: The tachyon potential $V(t)/M$ at level 0 (dotted line), level 3 (dashed line) and level 4 (full line).

much is known about the general convergence properties of level-truncation calculations in superstring theory. It would be nice to have a deeper understanding of this. Another interesting problem would be to study the interactions of the massless vector using Berkovits' action and compare with the different proposals for the non-abelian Dirac-Born-Infeld action [8].

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A. The conformal transformations of the fields

We now list the conformal transformations of the fields used in the calculation of the tachyon potential. To shorten the notation we denote $w = f(z)$.

$$\begin{aligned}
f \circ T(z) &= (f'(z))^{-1/2} T(w) \\
f \circ A(z) &= f'(z) A(w) - \frac{f''(z)}{f'(z)} c\partial c \, \xi \partial \xi \, e^{-2\phi}(w) \\
f \circ E(z) &= f'(z) E(w) - \frac{f''(z)}{2 f'(z)} \\
f \circ F(z) &= f'(z) F(w)
\end{aligned}$$

$$
\begin{aligned}
f\circ K(z) &= (f'(z))^{3/2}K(w) + 2\frac{f''(z)}{f'(z)}(f'(z))^{1/2}\xi c\partial\left(e^{-\phi}\right)(w) + \\
&+ \left[\frac{1}{2}\frac{f'''}{f'} - \frac{1}{4}\left(\frac{f''}{f'}\right)^2\right](f'(z))^{-1/2}\xi c e^{-\phi}(w) \\
f\circ L(z) &= (f'(z))^{3/2}L(w) + \frac{f''(z)}{f'(z)}(f'(z))^{1/2}\xi c\partial\phi\; e^{-\phi}(w) + \\
&+ \left[\frac{3}{4}\left(\frac{f''}{f'}\right)^2 - \frac{2}{3}\frac{f'''}{f'}\right](f'(z))^{-1/2}\xi c e^{-\phi}(w) \\
f\circ M(z) &= (f'(z))^{3/2}M(w) + \frac{15}{12}\left[\frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2\right](f'(z))^{-1/2}\xi c e^{-\phi}(w) \\
f\circ N(z) &= (f'(z))^{3/2}N(w) - \frac{f''(z)}{f'(z)}(f'(z))^{1/2}\xi\partial c\; e^{-\phi}(w) + \\
&+ \left[2\left(\frac{f''}{f'}\right)^2 - \frac{f'''}{f'}\right](f'(z))^{-1/2}\xi c\; e^{-\phi}(w) \\
f\circ P(z) &= (f'(z))^{3/2}P(w) + \frac{1}{2}\frac{f''(z)}{f'(z)}(f'(z))^{1/2}\partial\xi\; c e^{-\phi}(w) + \\
&+ \left[\frac{1}{4}\left(\frac{f''}{f'}\right)^2 - \frac{1}{6}\frac{f'''}{f'}\right](f'(z))^{-1/2}\xi c e^{-\phi}(w)
\end{aligned}
$$

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